MATH 5A - TEST
Spring 2022
(Chapter 3.9, 4 & 5.1)

100 POINTS

NAME: ______________________

Show your work on this paper.

(1) Evaluate the following integrals. (3 points each)

(a) $\int \cos 6x \, dx$

(b) $\int \frac{1}{x^3} \, dx$

(c) $\int 4\sec^2 z \, dz$

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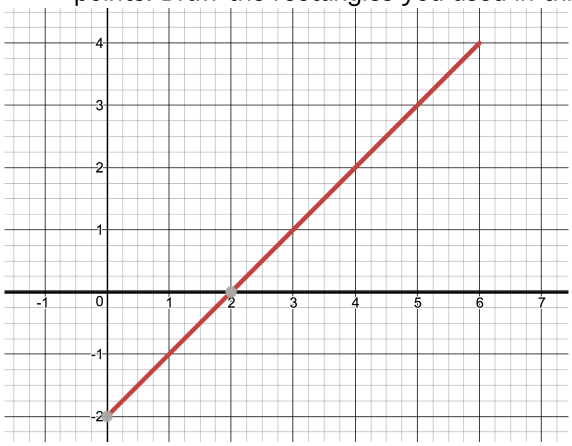
(2) Find the derivative of the function g(x) = $\int_2^{x^2} \sin^3 t \, dt$ (3 points)

(3) In this problem you will evaluate $\int_0^6 (x-2)\,dx$ using the 4 methods discussed in class. (19 points)

a) Estimate the value of $\int_0^6 (x-2)\,dx$ using n= 6 subintervals and using the left endpoints as sample points. Draw the rectangles you used in this approximation.



b) Find the exact value using the Riemann sum definition with sample points being right endpoints and the fact that $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$

c) Compute $\int_0^6 (x-2)\,dx$ using the area interpretation.

d) Compute $\int_0^6 (x-2)\,dx$ using the FTC part 2 and the antiderivative.

(4) Evaluate the following integrals. Give simplified, exact answers. (7 points each)

(a) On this problem only, you MUST make a u-substitution and change to U's limits. On subsequent definite integrals you can choose to switch to u's limits or not, but use proper notation.

$$\int_0^{\pi/2} \cos x \sqrt{\sin x}\ dx$$

(b) $$\int \frac{\sqrt{t} - 7t^2}{t^2}\, dt$$

(c) $$\int_{2/3}^{3} \frac{1}{\sqrt[3]{1-3x}}\, dx$$

(d) $$\int_{-1}^{3} \left(5x - |x|\right) dx$$

(4 continued)

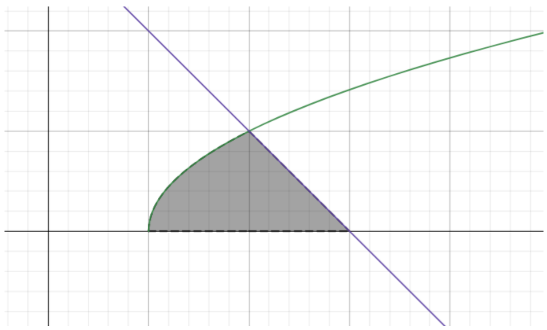
(e) $$\int \frac{\cos\left(\frac{1}{x}\right)}{3x^2}\,dx$$

(f) $$\int x^3\sqrt{x^2+1}\,dx$$

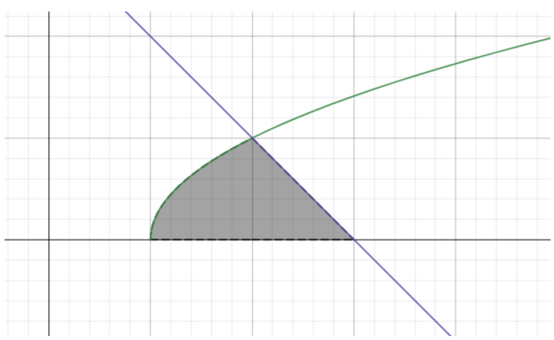
(g) $$\int_{-1}^{1} \frac{x}{\sqrt[3]{1+x^2}}\,dx$$

(6) Given the region bounded by the graphs of $y = \sqrt{x-1}$; $y = 3 - x$, and the x axis (12 points)

(a) Set up, but do not evaluate, an integral expression to find the area by integrating with respect to x.

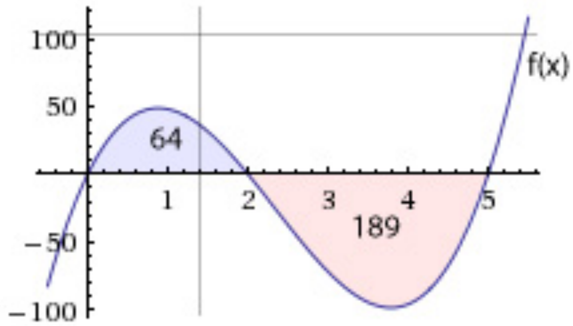


(b) Set up, but do not evaluate, an integral expression to find the area by integrating with respect to y.



(c) Find the area by evaluating one of the above.

(7) (8 points)



(a) Given the graph of y=f(x) and the <u>areas</u> shown in the figure above, find the following.

$$\int_0^2 f(x)dx = \_\_\_\_\_\_\_ \quad \int_2^5 f(x)dx = \_\_\_\_\_\_\_\_ \quad \int_0^5 f(x)dx = \_\_\_\_\_\_.$$

(b) Write an integral expression in terms of f(x) which would give the total enclosed area.